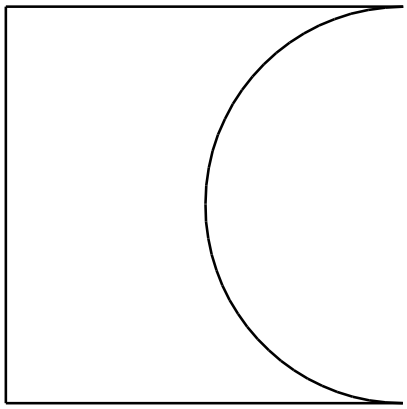
PART A Mathematics

1 A square piece of metal has a semicircular piece cut out of it as shown. The area of the remaining metal is $100\,\text{cm}^2$.



[diagram not to scale]

Which one of the following is a correct expression for the length of the side of the square in centimetres?

A $20\sqrt{\dfrac{2}{8-\pi}}$

B $10\sqrt{\dfrac{2}{4-\pi}}$

C $20\sqrt{\dfrac{2}{8+\pi}}$

D $10\sqrt{\dfrac{1}{8-\pi}}$

E $20\sqrt{\dfrac{1}{4-\pi}}$

2 In a right-angled triangle PQR the hypotenuse is the side PR.

The length of side PQ is 20 cm and the ratio RQ:PQ is 1:2

What is the length of the perpendicular from the hypotenuse to the point Q?

A $8\sqrt{5}$ cm

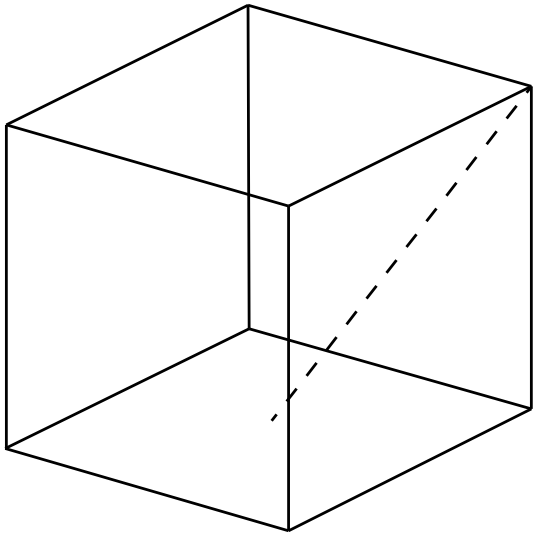
B $10\sqrt{2}$ cm

C $2\sqrt{5}$ cm

D $5\sqrt{2}$ cm

E $4\sqrt{5}$ cm

3 A cube has sides of unit length. What is the length of a line joining a vertex to the midpoint of one of the opposite faces (the dashed line in the diagram below)?



[diagram not to scale]

A $\sqrt{\frac{3}{2}}$

B $\sqrt{2}$

C $\sqrt{\frac{5}{2}}$

D $\sqrt{3}$

E $\sqrt{5}$

4 If you look at a clock and the time is 9.45, what is the angle between the hour and the minute hands?

A 0°

B 7.5°

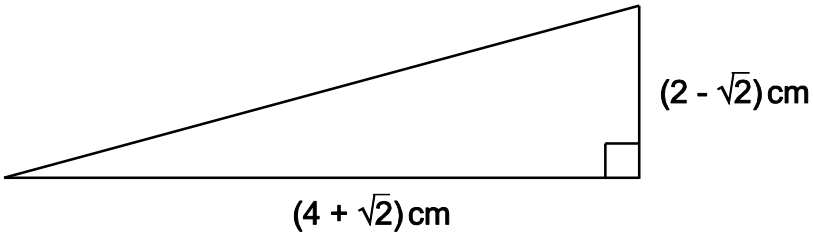
C 15°

D 22.5°

E 30°

5 The right-angled triangle shown has horizontal and vertical sides measuring $(4+\sqrt{2})$ cm and $(2-\sqrt{2})$ cm respectively.

[diagram not to scale]



Calculate the area of the triangle.

A $(5+3\sqrt{2})\,\text{cm}^2$

B $(3-\sqrt{2})\,\text{cm}^2$

C $(3+3\sqrt{2})\,\text{cm}^2$

D $(5-\sqrt{2})\,\text{cm}^2$

6 A solid sphere of radius *r* fits inside a hollow cylinder. The cylinder has the same internal diameter and length as the diameter of the sphere.

The volume of a sphere is $\frac{4}{3}\pi r^3$, where *r* is the radius of the sphere.

What fraction of the space inside the cylinder is taken up by the sphere?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

7 Which of the expressions below has the largest value for $0 < x < 1$?

A $\frac{1}{x}$

B x^2

C $\frac{1}{(1+x)}$

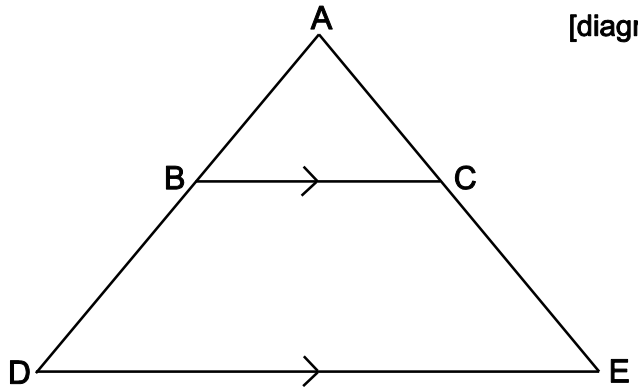
D $\frac{1}{\sqrt{x}}$

E $\sqrt{x}$

8 A shape is formed by drawing a triangle ABC inside the triangle ADE.

BC is parallel to DE.

AB $= 4$ cm BC $= x$ cm DE $= x + 3$ cm DB $= x - 4$ cm



[diagram not to scale]

Calculate the length of DE.

A 5 cm

B 7 cm

C 9 cm

D $4 + 2\sqrt{7}$ cm

E $7 + 2\sqrt{7}$ cm

9 Two variables are connected by the relation: $P \propto \frac{1}{Q^2}$

Q is increased by 40%.

To the nearest percent, describe the change in P in percentage terms.

A 29% decrease

B 44% decrease

C 49% decrease

D 51% decrease

E 80% decrease

F 96% decrease

10 Three variables x, y and z are known to be related to each other in the following ways:

x is directly proportional to the square of z.
y is inversely proportional to the cube of z.

Which of the following correctly describes the relationship between x and y?

A The square of x is directly proportional to the cube of y.

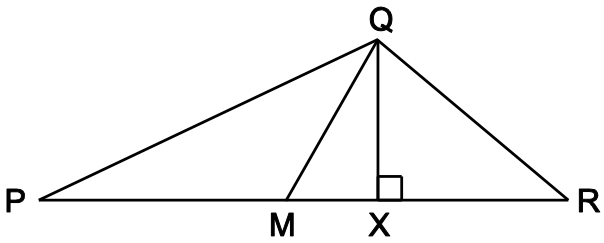
B The square of x is inversely proportional to the cube of y.

C The cube of x is directly proportional to the square of y.

D The cube of x is inversely proportional to the square of y.

E x is directly proportional to y^6.

11 In the triangle PQR shown below:



[diagram not to scale]

X lies on PR

$\angle$QXR is 90°

$\frac{QX}{PX} = \frac{1}{6}$

$\frac{QX}{XR} = \frac{2}{3}$

M is the midpoint of PR.

What is $\frac{QX}{MX}$?

A $\frac{1}{9}$

B $\frac{5}{12}$

C $\frac{4}{9}$

D $\frac{1}{2}$

E $\frac{5}{6}$

12 Solve the inequality $x^2 \geq 8 - 2x$

A $x \geq 4$

B $x \leq 2$ and $x \geq -4$

C $x \geq -2$ and $x \leq 4$

D $x \geq 2$ or $x \leq -4$

13 The total surface area of a cylinder, measured in square centimetres, is numerically the same as its volume, measured in cubic centimetres.

The radius of the cylinder is r cm, the height is h cm.

Express h in terms of r.

A $h = \dfrac{2r}{r-2}$

B $h = \dfrac{2r}{r+2}$

C $h = r+2$

D $h = r-2$

E $h = 2r(r-2)$

14 How many different integers, n, are there such that the difference between $2\sqrt{n}$ and 7 is less than 1?

A 0

B 2

C 4

D 6

E 8

15 The square PQRS is positioned so that its vertices are at the points with coordinates: (1, 1), $(-1, 1)$, $(-1, -1)$ and $(1, -1)$.

The square is rotated clockwise through 90° about the origin and then reflected in the line $y = x$.

Which transformation will return the square to its original orientation?

A A reflection in the x-axis.

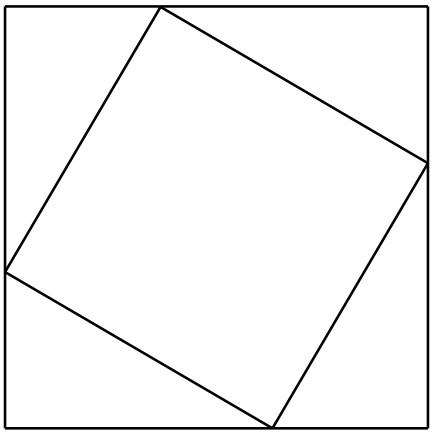
B A reflection in the y-axis.

C A reflection in the line $y = -x$.

D A rotation of 90° clockwise about the origin.

E A rotation of 90° anticlockwise about the origin.

16 A design is set up by joining the points which are one third of the way along the sides of a square. This forms a second square as shown.



[diagram not to scale]

This process is repeated.

Calculate the area of the fourth square as a fraction of the original square.

A $\frac{1}{27}$

B $\frac{5\sqrt{5}}{81}$

C $\frac{125}{729}$

D $\frac{25}{81}$

E $\frac{5\sqrt{5}}{27}$

17 The equation connects the variables M, x, y, z, P and Q.

$$M = \frac{(x+y)^2 z}{P} Q$$

The following changes are made:

x and y are both increased by 50%

z is decreased by 20%

P is doubled and Q remains the same.

What is the resulting percentage change in M?

A 2.5% decrease

B 2.5% increase

C 10% decrease

D 10% increase

E 20% decrease

F 20% increase

18 I have two six-sided dice, each with faces numbered from 1 to 6. One of the dice is fair, but the other is not; it will land on numbers 1 to 5 with equal probability, but lands on 6 with a different probability.

When I roll the dice the probability that I get a total of 12 is $\frac{1}{18}$.

What is the probability that I get a total of 2 when I roll the dice?

A $\frac{1}{72}$

B $\frac{1}{45}$

C $\frac{1}{36}$

D $\frac{1}{18}$

E $\frac{1}{9}$